Model Question paper of B.Sc Sem IV
Mat C-8 - Analysis II

Short answer type Questions.

(1) Define Comparision test for the Convergence of $\int_c^\infty f(x)\,dx$. and test the Convergence of the integral $\int_0^\infty \frac{\cos x}{1+x^2}\,dx$.

(2) Test the Convergence of $\int_a^\infty e^{-x} \frac{\sin x}{x^2}\,dx$ where $a>0$.

(3) Show that $\int_0^\infty \sin x^2\,dx$ is Convergent.

(4) Evaluate Gamma function

$$\Gamma n = \int_0^\infty x^{n-1} e^{-x}\,dx.$$

(5) Prove that $B(l,m) = \frac{\Gamma(l)\,\Gamma(m)}{\Gamma(l+m)}$

(6) $\int_0^\infty \frac{x\,dx}{1+x^6}$

(7) Transform $\iint v\, dx\, dy$ to polar Co-ordinates.

(8) Transform the multiple integral $\iiint v\, dx\, dy\, dz$ by the Polar transformations.

(9) Write the statement of Liouville's Extension of Dirichlet's Theorem & Evaluate $\iiint xyz\, dx\, dy\, dz$ taking throughout the ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} \leq 1 .$$

(10) Evaluate $\int_C F \cdot dr$, where $F = (x^2 - y^2)\vec{i} + xy\vec{j}$ and Curve C is the arc of the curve $y = x^3$ from $(0,0)$ to $(2,8)$.

(11) State and prove Green's Theorem.

(12) If $F = ax\vec{i} + by\vec{j} + cz\vec{k}$, a, b, c are Constants, show that $\iint_S F \cdot n\, ds = \frac{4}{3}\pi(a+b+c)$, where S is the surface of a unit sphere.

(13) Evaluate by Green's theorem

$$\oint_C (x^2 - \cosh y)\,dx + (y + \sin x)\,dy .$$

(14) Find the work done when a force $F = (x^2 - y^2 + x)\vec{i} - (2xy + y)\vec{j}$ moves a particle in xy plane from $(0,0)$ to $(1,1)$ along the parabola $y^2 = x$.

(15) If S is any closed surface enclosing a volume V and $F = x\vec{i} + 2y\vec{j} + 3zk$, prove that

$$\iint F \cdot n \, dS = 6V .$$

Long Answer Type Questions.

1> Show that the integral

$$\int_0^\infty e^{-ax} \frac{\sin x}{x}\,dx$$ is Convergent when $\alpha \geq 0$.

(2) Discuss the Convergence of Beta function

$$\int_0^1 x^{m-1}(1-x)^{n-1}\,dx$$

(3) Discuss the Convergence of Gamma function

$$\int_0^\infty x^{n-1} e^{-x}\, dx .$$

(4) prove that (Duplication Formula)

$$\Gamma(m)\,\Gamma\left(m+\tfrac{1}{2}\right) = \frac{\sqrt{\pi}}{2^{2m-1}}\,\Gamma(2m).$$

(5) Show that

$$\int_0^{\pi/2} \frac{dx}{\sqrt{\sin x}} \times \int_0^{\pi/2} \sqrt{\sin x}\, dx = \pi$$

(6) verify Green's theorem in the plane for

$$\oint_C (xy+y^2)\,dx + x^2\,dy \quad \text{where } C$$

is the closed Curve of the region bounded by $y = x$ & $y = x^2$.

(7) verify divergence theorem for

$$F = (x^2 - yz)\vec{i} + (y^2 - zx)\vec{j} + (z^2 - xy)\vec{k}$$

taken over the rectangular parallelopiped

$0 \le x \le a, \quad 0 \le y \le b, \quad 0 \le z \le c$.

(8) State & prove Stoke's Theorem.

(9) Verify Stoke's theorem for

$$F = (x^2+y^2)\vec{i} - 2xy\vec{j}$$

taken round the rectangle bounded by

$$x = \pm a, \quad y = 0, \quad y = b.$$

(10) Verify Stoke's Theorem for $F = -y^3\vec{i} + x^3\vec{j}$, where S is the circular disc $x^2+y^2 \le 1$, $z = 0$.